Sanjay Ghodawat University, Kolhapur

Established as State Private University under Govt. of Maharashtra. Act No XL, 2017

2018-19
EXM/P/09/01

Year and Program:2018-19 M.Sc.-I | **School of Science** | **Department of Mathematics**

Course Code: MTS505 | **Course Title: Topology** | **Semester – I**

Day and Date Friday 30/10/18 | **End Semester Examination (ESE)** | **Time:** 10:00 to 10:30 am | **Max Marks: 100**

Student's Signature: | **Answer Booklet No.:**

PRN/Exam Seat No.: | **Invigilator's Signature:**

Instructions:

1) All questions are compulsory.
2) **Attempt Q.1 within first 30 minutes.**
3) Each MCQ type question has four choices out of which only one is correct.
4) Tick mark (√) the correct alternative which should be answered in question paper itself and submit to the Jr. Supervisor.
5) If you tick more than one option, then it will not be evaluated.
6) Figures to the right indicate full marks.
7) Use **Blue ball pen** only.

Q.1	**Tick the correct option.**	**Marks**	**Bloom's Level**	**CO**
i.	Let X be finite set and $\tau = \{\emptyset\} \cup \{A \subseteq X, A^c \text{ is finite set}\}$,then this topology coincides with _____ A) Discrete topology B) Indiscrete topology C) X D) $\emptyset$	02	L_2	CO1
ii.	In any topological space (X, τ), $\emptyset$ and X are always_____. A)Closed B) Open C)Closed and open D)Neither open nor closed	02	L_1	CO1
iii.	If $A \subseteq X$, then boundary of A i. e. $b(A)$ is always _____. A)Closed B) Open C)Closed and open D)Neither open nor closed	02	L_2	CO2

iv. In a topological space (X, τ), consider the following statements 02 L_2 CO2

I) A set is open iff it is neighborhood of each of its points.

II) Intersection of two neighborhoods of x is again a neighborhood of x, where $x \in X$.

A) Only I is true B) Only II is true

C) Both I and II are true D) Both I and II are false.

v. Let (X, τ_1) and (Y, τ_2) be two topological spaces. The mapping $f: X \to Y$ is continuous iff_____. 02 L_2 CO3

A) Image of every open set is open.

B) Image of every closed set is open.

C) Inverse image of every open set is open.

D) Inverse image of every closed set is open.

vi. Let (X, τ_1) and (Y, τ_2) be two topological spaces. The mapping $f: X \to Y$ is continuous iff_____ 02 L_1 CO3

A) $f[c(A)] \subseteq c^*[f(A)]$, $\forall A \subseteq X$

B) $c^*[f(A)] \subseteq f[c(A)]$, $\forall A \subseteq X$

C) $f(A) \subseteq c^*[f(A)]$, $\forall A \subseteq X$

D) $c^*[f(A)] \subseteq f(A)$, $\forall A \subseteq X$

vii. Let (X, τ) be any topological space. If every subspace of (X, τ) has a property same as that of its space(X, τ), then that property is called as _____. 02 L_1 CO4

A)Finite Intersection Property B)Topological Property

C)Hereditary Property D)None

viii. If a subset E of real line R is connected, then E is always _____. 02 L_2 CO4

A)a union of two disjoint intervals B)an interval

C) a singleton set D)an empty set.

ix. If (X, τ) is a T_0- space, then closure of distinct points of X are always_____. 02 L_1 CO5

A)Identical B)Equal to $\emptyset$ C)Distinct D)equal to X

x. If (X, τ) is a discrete topological space with $|X| \geq 2$,then this space is_____. 02 L_2 CO5

A) T_0- space B) T_1- space C) T_2- space D)All of these.

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Course Code: MTS505 | **Course Title: Topology** | **Semester – I**

Day and Date Friday 30 Nov 18 | **End Semester Examination (ESE)** | **Time:** 10:30 to 1:00 pm | **Max Marks: 100**

Instructions:
1) All questions are compulsory.
2) Non-programmable calculator is allowed.
3) Figures to the right indicate full marks.

			Marks	Bloom's Level	CO
Q.2	a)	Let $\tau_1 = \{\emptyset, R\} \cup \{L_a : a \in R\}$,where $L_a = \{x \in R : x < a\}$, and $\tau_2 = \{\emptyset, R\} \cup \{R_a : a \in R\}$,where $R_a = \{x \in R : x > a\}$, then show that τ_1 and τ_2 are topologies on R ,where R is the set of real numbers.	10	L_4	CO1
		OR			
	a)	Define derived set. Also if X=$\{a, b, c\}$ and $\tau = \{\emptyset, X, \{a\}, \{b\}, \{a, b\}\}$,then find d $(\{a, b\})$.	10	L_3	CO1
Q.3	a)	Define basis. Let $X = \{a,b,c\}$, $\tau = \{\emptyset, X, \{a\}, \{b, c\}, \{a, c\}, \{c\}, \{a\}\}$. Then which of the following subfamilies form basis? Also mention the valid reason for each subfamily. $B_1 = \{\{a\}, \{b\}, \{c\}\}$, $B_2 = \{\{a\}, \{b, c\}, \{c\}\}$, $B_3 = \{\{a\}, \{b\}\}$, $B_4 = \{\{a\}, \{a, b\}, \{b\}, \{c\}\}$.	10	L_3	CO2
		OR			
	a)	Let (X, τ) be a topological space and let Y be a non-empty subset of X. If $\tau^* = \{G \cap Y : G \in \tau\}$, then show that τ^* is topology on Y. Also show that the set F^* is closed in Y if and only if there exist a set F closed in X such that $F^* = F \cap Y$.	10	L_4	CO2
Q.4	a)	Define the following: i) Open mapping ii) Closed mapping iii) Homeomorphism	03	L_2	CO3
	b)	Let (X, τ_1) and (Y, τ_2) be two topological spaces. Then prove that $f : X \to Y$ is continuous iff inverse image of every τ_2-open set is τ_1-open set.	07	L_4	CO3
		OR			
	b)	If (X, τ) and (X^*, τ^*) are two topological spaces, then prove that a mapping $f : X \to X^*$ is closed if and only if $c^*[f(A)] \subseteq f[c(A)]$.	07	L_4	CO3

Q.5	a)	Let E be a subset of subspace (Y, τ^*) of a topological space (X, τ), then prove that E is τ^* connected if and only if E is τ connected.	07	L_4	CO4
		OR			
	a)	State and prove Heine-Borel Theorem.	07	L_4	CO4
	b)	Solve any **THREE**			CO4
		i) Prove that every closed subset of a compact topological space is compact.	06	L_3	CO4
		ii) Show that the usual topological space $(\mathbb{R}, \tau_u)$ is not compact.	06	L_4	CO4
		iii) Prove that the two closed subsets A and B of a topological space (X, τ) are separated, if they are disjoint.	06	L_4	CO4
		iv) If every two points of the set E are contained in some connected subset of E, then prove that E is connected.	06	L_6	CO4
Q.6		Attempt any **FIVE**			
	a)	Show that the usual topological space $(\mathbb{R}, \tau_u)$ is T_0- space.	05	L_4	CO5
	b)	Show that the property of space being T_1- space is preserved under one-to-one, onto and open maps.	05	L_4	CO5
	c)	Prove that a topological space (X, τ) is T_1- space if and only if every finite subset of X is closed.	05	L_4	CO5
	d)	Prove that every continuous mapping of a compact space into a Hausdorff space is closed mapping.	05	L_4	CO5
	e)	Prove that being T_2- space is a hereditary property.	05	L_6	CO5
	f)	Let (X, τ) be any topological space. If any two disjoint compact subsets of X can be separated by disjoint open sets, then prove that (X, τ) is a T_2- space	05	L_4	CO5

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