Sanjay Ghodawat University, Kolhapur

Established as State Private University under Govt. of Maharashtra. Act No XL, 2017

2018-19
EXM/P/09/01

Year and Program:2018-19 M.Sc.I | **School of Science** | **Department : Mathematics**

Course Code: MTS 509 | **Course Title: Ordinary Differential Equation** | **Semester – I**

Day and Date: Wednesday 05 Dec 18 | **End Semester Examination (ESE)** | **Time:** 10:00 to 10:30 am

Max Marks: 100

PRN/Exam seat No: | **Answer booklet No:**

Student's signature: | **Invigilator signature:**

Instructions:

1) All questions are compulsory.
2) **Attempt Q.1 within first 30 minutes.**
3) Each MCQ type question has four choices out of which only one is correct
4) Tick mark (√) the correct alternativewhich should be answered in question paper itself and submit to the Jr.Supervisor.
5) If you tick more than one option it will not be evaluated
6) Figures to the right indicate full marks
7) Use **Blue ball pen** only.

		Marks	Bloom's Level	CO
Q.1	**Select the correct alternative.**			
i	If ϕ_1 and ϕ_2 be two linearly independent solution of $L(y)=0$ then A) $W(\phi_1,\phi_2)(x)=0$ B) $W(\phi_1,\phi_2)(x)\neq 0$ C) $W(\phi_1,\phi_2)(x)=1$ D) $W(\phi_1,\phi_2)(x)\neq 1$	02	L_2	CO1
ii	A solution of differential equation which contains no arbitrary constants is called A) Particular solution B) general solution C) primitive D) None of these	02	L_2	CO1
iii	The two linearly independent solutions of $y''-\frac{2}{x^2}y=0$ A) $\phi_1(x)=x, \phi_2(x)=\frac{1}{x^2}$ B) $\phi_1(x)=x, \phi_2(x)=\frac{1}{x}$ C) $\phi_1(x)=x^2, \phi_2(x)=\frac{1}{x}$ D) $\phi_1(x)=2x, \phi_2(x)=x^{\frac{1}{2}}$	02	L_3	CO2

iv	Consider the statement I) Let ϕ be a solution of L(y)= $y^n + a_1 y^{n-1} + ... + a_n y$, where $\phi_1(x) \neq$ then we can reduce the order of equation by one if $v_2, v_3, ... v_n$ are independent solutions. II)For homogenous linear ordinary differential equation if $\phi_1, \phi_2 ....$ be solutions then $c_1\phi_1 + c_2\phi_2 + ... + c_n\phi_n$ is again a solution. A) I and II True B) I and II False C) Only I True D) Only II True	02	L_2	CO2
v	The singular points of equation $y'' + xy = 0$ are..... A) 0 and 1 B) 1 and 2 C) there are no singular points D)cannot be determined	02	L_3	CO3
vi	$(x^{-\alpha} J_\alpha)'(x) = ...$ A) $x^\alpha J_\alpha(x)$ B) $x^\alpha J_{\alpha+1}(x)$ C) $-x^{-\alpha} J_{\alpha+1}(x)$ D) $x^{\alpha+1} J_{\alpha+1}(x)$	02	L_3	CO3
vii	Let f is a function defined for (x, y) in a set S then f satisfies Lipchitz condition on S if $K > 0$ such that ..... A) $\lvert f(x, y_1) - f(x, y_2) \rvert \leq K \lvert y_1 + y_2 \rvert$ B) $\lvert f(x, y_1) - f(x, y_2) \rvert \leq K \lvert y_1 - y_2 \rvert$ C) $\lvert f(x, y_1) - f(x, y_2) \rvert \leq K \lvert y_1 . y_2 \rvert$ D) None of these	02	L_2	CO4
vii	The Lipchitz constant of the function $f(x, y) = x^2 \lvert y \rvert$ is A) 2 B) -1 C) 1 D) -2	02	L_3	CO4
ix	The set of all eigen values of regular strum Liouville's Boundary value problem is called ..... A) Eigen space B) space C) Eigen field D) spectrum	02	L_2	CO5
x	The eigen values of a regular strum Liouville's boundary value problem are ..... A) always Complex B) may be complex or real C) always Real D) Cannot define	02	L_2	CO5

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Course Code: MTS 509	**Course Title: Ordinary Differential Equation**	**Semester – I**
Day and Date: Wednesday 05 Dec 18	**End Semester Examination (ESE)**	**Time:** 10:00 to 1:00 pm **Max Marks: 100**

Instructions: 1) All questions are compulsory.
2) Figures to the right indicate full marks.

		Marks	Bloom's Level	CO
Q.2	**Solve the following.**			
a)	Find the solution of $y''-4y'+5y=0$	04	L_3	CO1
b)	Show that every solution of the constant coefficient equation $y''+a_1y'+a_2y=0$ tends to zero as $x\to\infty$ if and only if the real parts of the characteristic polynomial are negative.	06	L_3	CO1
	OR			
b)	Determine whether the following functions are linearly independent or Linearly dependent? i) $\phi_1(x)=x$, $\phi_2(x)=e^{rx}$, where r is complex constant ii) $\phi_1(x)=\cos x$, $\phi_2(x)=\sin x$	06	L_3	CO1
Q.3	**Solve any Two.**			
a)	If $\phi_1(x)$ is a solution of $L(y)=y''+a_1y'+a_2y=0$ on an interval I and $\phi_1(x)\neq 0$ on I then the second solution $\phi_2(x)$ is given by $\phi_2(x)=\phi_1(x)\int_{x_0}^{x}\frac{1}{[\phi_1(s)]^2}\exp(-\int_{x_0}^{s}a_1(t)dt)\ ds$	05	L_3	CO2
b)	If one solution of $x^2y''-7xy'+15y=0$ is $\phi_1(x)=x^3\ (x>0)$ then verify that the function $\phi_1(x)$ satisfies the equation and also find another independent solution.	05	L_3	CO2
c)	If one solution of $y''-\frac{2}{x^2}y=0\ (0<x<\infty)$ is $\phi_1(x)=x^2$, then find another independent solution.	05	L_3	CO2
Q.4	**Solve any one**			
a)	Find two Linearly Independent power series solution(in power of x) of $y''-x^2y=0$	10	L_3	CO3

	b)	Show that i) $J'_0(x) = -J_1(x)$ ii) $(x^\alpha J_\alpha)'(x) = x^\alpha J_{\alpha-1}(x)$	10	L_3	CO3
Q.5		**Solve the following**			
	a)	Show that $f(x,y) = xy^2$ satisfies Lipchitz condition on R for $\lvert x\rvert \le 1$ & $\lvert y\rvert \le 1$ which does not satisfies on the strip $\{S : \lvert x\rvert \le 1, \lvert y\rvert \le \infty\}$	06	L_3	CO4
	b)	Calculate first four successive approximations $\phi_0, \phi_1, \phi_2, \phi_3$ for $y' = x^2 + y^2$ with $y(0) = 0$	07	L_3	CO4
		OR			
	b)	Consider $y' = x^2 + y^2$, $y(0) = 0$ on $\{R : \lvert x\rvert \le 1, \lvert y\rvert \le 1\}$ .Compute an upper bound *M* for *f(x,y)* on R. Also find the interval containing *x*=0 with all the successive approximations exist in R.	07	L_3	CO4
	c)	Let f be a continuous real valued function on the rectangle $R = \{(x,y) \mid \lvert x - x_0\rvert \le a \lvert y - y_0\rvert \le b\}$ for a,b>0and $\lvert f(x,y)\rvert \le M$ $\forall (x,y) \in R$ .Suppose *f* satisfies Lipchitz condition with Lipchitz constant in R then show that the successive approximations $\phi_0(x) = y_0$, $\phi_{k+1}(x) = y_0 + \int_{x_0}^{x} f(t, \phi_k(t))dt$ converges on $I = \{x \mid \lvert x - x_0\rvert \le \alpha\}$, where $\alpha = \min\left(a, \frac{b}{m}\right)$, to a solution ϕ of the initial value problem $y' = f(x,y)$; $y(x_0) = y_0$	12	L_4	CO4
		OR			
	c)	Prove that a function ϕ is the solution of the initial value problem $y' = f(x,y)$, $y(x_0) = y_0$ on an interval I if and only if it is a solution of the integral equation $y = y_0 + \int_{x_0}^{x} f(t,y)dt$ on I. Also show that the function *f* given by $f(x,y) = x^2\lvert y\rvert$ satisfies Lipchitz condition on $R = \{(x,y) \mid \lvert x\rvert \le 1, \lvert y\rvert \le 1\}$ with the condition $\frac{\partial f}{\partial y}$ does not exist at $(x,0)$ if $x \ne 0$	12	L_4	CO4

Q.6

a)	Construct Green's function for initial value problem of the differential equation $u''=0$	05	L_4	CO5
b)	**Attempt Any Two.**			
	I) Let the function $G(t,\tau)$ be defined as follows $G(t,\tau)=0$ for $a \le t \le \tau$ for each fixed $\tau \ge a$ and all $t > \tau$ .If$G(t,\tau)$ is solution of differential equation $L(G)=G_{tt}+p(t)G_t+q(t)G=0$ which satisfies initial conditions G=0 $G_t=1$ at $t=\tau$, then show that G isGreen'sfunction of the second order differential equation.	10	L_4	CO5
	II) Let u and v be two Eigen functions of strum Liouville's boundary value problem $\frac{d}{dx}\left[p(x)\frac{du}{dx}\right]+q(x)u(x)+\lambda r(x)u(x)=0$ for $a \le x \le b$ with boundary conditions $\alpha_1 u(a)+\beta_1 u'(a)=0$ & $\alpha_2 u(b)+\beta_2 u'(b)=0$ corresponding to Eigen values λ and μ then prove that $\left\{p(x).\ W\left(u(x),v(x)\right)\right\}_a^b=0$, where $W\left(u(x),v(x)\right)$ is Wronskian of u and v.	10	L_4	CO5
	III) Integrate the following differential equation by constructing Green's function $4y''-16y'+15y=x$	10	L_3	CO5

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