Sanjay Ghodawat University, Kolhapur

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2018-19
EXM/P/09/00

Year and Program:	**School of Science**	**Department of Mathematics**
2018-19 F.Y. M. Sc.		
Course Code – MTS 507	**Course Title – Numerical Analysis**	**Semester – I**
Day and Date –	**Examination: (ESE) End Semester Examination**	**Max Marks: 100**

Student's Signature: **Invigilator's Signature:**

Seat Number: **PRN No.:** **Answer Book No.:**

Instructions:

1) All questions are compulsory.
2) **Attempt Q.1 within first 30 minutes.**
3) Each MCQ type question has four choices out of which only one is the correct.
4) Tick mark (√) the correct alternative which should be answered in question paper itself and submit to the Jr. Supervisor.
5) If you tick more than one option it will not be evaluated.
6) Figures to the right indicate full marks.
7) Use **Blue ball pen** only.

Q.1	**Attempt the following questions.**	Marks	Bloom's Level	COs
i)	The rate of convergence of Secant method is A. 1.618 B. 1.186 C. 1.816 D. 1.861	2	L_2	CO1
ii)	Newton-Raphson iteration formula for finding $\sqrt[3]{c}$, where c >0 is A. $x_{n+1} = \frac{2x_n^3 + \sqrt[3]{c}}{2x_n^2}$ B. $x_{n+1} = \frac{3x_n^3 - \sqrt[3]{c}}{3x_n^2}$ C. $x_{n+1} = \frac{2x_n^3 + c}{3x_n^2}$ D. $x_{n+1} = \frac{2x_n^3 - c}{3x_n^2}$	2	L_3	CO1

iii) In which of the following method, symmetric matrix A is reduced to diagonal matrix. 2 L_2 CO2

A. Householder method for symmetric matrices.
B. Jacobi method for symmetric matrices.
C. Doolittle's method.
D. None of these.

iv) For the matrix $\begin{bmatrix} 0 & 1 & -1 & 1 \\ 1 & 0 & 0 & 2 \\ -1 & 0 & 0 & 0 \\ 1 & 2 & 0 & 0 \end{bmatrix}$, the bound for the Eigen values predicted by Gershgorin's theorem is 2 L_3 CO2

A. 3
B. 1
C. 2
D. 4

v) Expression for the function $x^3 + x^2 - 5x + 2$ in factorial notation is 2 L_3 CO3

A. $x^{(3)} + 4x^{(2)} - 3x^{(1)} + 2$
B. $x^{(3)} - 4x^{(2)} - 3x^{(1)} + 2$
C. $x^{(3)} - 4x^{(2)} + 3x^{(1)} + 2$
D. $x^{(3)} + 4x^{(2)} - 3x^{(1)} - 2$

vi) Let E_1 be the error for a linear interpolation of a equispaced data 2 L_2 CO3

x	x_0	x_1
f(x)	$f(x_0)$	$f(x_1)$

then

A. $|E_1| \leq \frac{1}{2}$
B. $|E_1| \leq \frac{1}{4}$
C. $|E_1| \leq \frac{1}{8}$
D. $|E_1| \geq \frac{1}{8}$

vii) Simpson's $\frac{1}{3}rd$ rule for evaluation of $\int_a^b f(x)dx$ requires the interval $[a, b]$ to be divided into 2 L_2 CO4

A. An even number of subintervals of equal width.
B. An odd number of subintervals of equal width.
C. Any number of subintervals of equal width.
D. Any number of subintervals.

viii) Considering six subintervals, the value of $\int_1^7 \frac{1}{x}dx$ by Simpson's $\frac{1}{3}rd$ rule is | 2 | L_3 | CO4

A. 1.358
B. 1.958
C. 1.625
D. 1.458

ix) Single step methods are --------- | 2 | L_2 | CO5

A. Euler, Adam, Milne
B. Euler, RK method, Milne
C. Euler, Modified Euler, RK method, Taylor
D. Euler, Milne, and Taylor

x) Consider the initial value problem $\frac{dy}{dx} = f(x, y)$, with $y(x_0) = y_0$. Let $y_1 = y_0 + w_1k_1 + 3k_2$ approximate the solution of the above IVP at $x_1 = x_0 + h$ with $k_1 = hf(x_0,\ y_0)$ and $k_2 = hf(x_0 + ah,\ y_0 + bk_1)$ and h being the step size, if the formula for y_1 yields a second order method, then the value of w_1, a, b are | 2 | L_3 | CO5

A. $-1, \frac{1}{6}, \frac{1}{6}$
B. $-2, \frac{1}{3}, \frac{1}{3}$
C. $-2, \frac{1}{6}, \frac{1}{6}$
D. $3, \frac{1}{3}, \frac{1}{3}$

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Course Code: MTS 507 | **Course Title: Numerical Analysis** | **Semester – I**

Day and Date | **End Semester Examination (ESE)** | **Time:** | **Max Marks: 100**

Instructions:
1) All questions are compulsory.
2) Use of non-programmable calculator is allowed.
3) Figures to the right indicate full marks.

		Marks	Bloom's Level	CO
Q.2	**Solve the following**			
a)	Explain the Regula-Falsi method to find roots of an equation $f(x)=0$ and hence write advantages and disadvantages of the Regula-Falsi method.	06	L_2	CO1
	OR			
a)	If $\phi(x)$ is a continuous function in some interval $[a,\ b]$ that contains root and $\lvert\phi'(x)\rvert \le k < 1$ in this interval, then for every choice of $x_0 \in [a,\ b]$, the sequence $\{x_n\}$ determined from $x_{n+1}=\phi(x_n)$, $n=0,\ 1,\ 2,\ 3,\ 4,\ldots$ converges to the root ξ of $x=\phi(x)$.	06	L_2	CO1
b)	Find roots of $xe^x-1=0$ using secant method.	04	L_3	CO1
	OR			
b)	Using Newton-Raphson method, find real root of $x^3-5x+3=0$.	04	L_3	CO1
Q.3	**Solve the following**			
a)	If A is strictly diagonally dominant matrix, then the Gauss-Seidel iteration scheme converges for any initial starting vector.	06	L_3	CO2
	OR			
a)	State and prove Brauer theorem.	06	L_3	CO2
b)	Solve the following system of equations by Gauss-Seidel method $2x-y=7,\ -x+2y-z=1,\ -y+2z=1$. Take the initial approximation as $\bar{x}^{(0)}=\bar{0}$ and perform two iterations.	04	L_3	CO2
	OR			
b)	Estimate the Eigen values and region of matrix $A=\begin{bmatrix}3 & 2 & 2\\ 2 & 5 & 2\\ 2 & 2 & 3\end{bmatrix}$.	04	L_3	CO2

Q.4 **Solve the following**

a) Derive the Newton's backward difference interpolation formula. 06 L_2 CO3

OR

a) Any polynomial $f(x)$ in x can be expressed in the factorial notation with same degree. 06 L_2 CO3

b) If $f(1)=4,\ f(2)=4,\ f(7)=5$ and $f(8)=4$, find $f(5)$ using Lagrange's interpolation formula. 04 L_3 CO3

OR

b) Using Newton's forward difference formula, find y at $x=4.4$ from the table 04 L_3 CO3

x	0	2	4	6	8	10	12
y	12	7	6	7	13	32	77

Q.5 **Solve the following**

a) Derive Newton-Cotes quadrature formula and use it to derive Simpson's $\frac{3}{8}th$ rule. 12 L_2 CO4

OR

a) Derive the formula to obtain numerical differentiation using 12 L_2 CO4
(i) Newton's forward difference formula.
(ii) Newton's backward difference formula.
(iii) Stirling's Central difference formula.

b) Using following data find x for which y is maximum and find the value of y at x maximum. 06 L_3 CO4

x	1.2	1.3	1.4	1.5	1.6
y	0.9320	0.9636	0.9855	0.9975	0.9996

OR

b) Find the first derivative of the function tabulated below at the point $x=5$ from the following table. 06 L_3 CO4

x	0	1	2	3	4	5	6
$f(x)$	0	2.5	8.5	15.5	24.5	36.5	50

c) Obtain the error in methods 07 L_2 CO4

(i) Trapezoidal rule (ii) Simpson's $\frac{1}{3}rd$ rule.

OR

c) Compute the value of definite integral $\int_{0.2}^{1.4} (\sin x - \log_e x + e^x)dx$ by 07 L_3 CO4

(i) Trapezoidal rule (ii) Simpson's $\frac{1}{3}rd$ rule.

Compare the errors with the exact value of the integration. Take $h = 0.2$.

Q.6 **Solve the following**

a) Explain the Adams-Bashforth predictor & corrector method to solve the differential equation $\frac{dy}{dx} = f(x, y)$ with $y(x_0) = y_0$. 12 L_2 CO5

OR

a) Derive the formula to solve differential equation $\frac{dy}{dx} = f(x, y)$ with $y(x_0) = y_0$ by using 12 L_2 CO5

(i) Milne-Simpson method

(ii) Runge-Kutta method of second order.

b) Using Picard's method of successive approximation, obtain a solution upto 5th approximation of the equation $\frac{dy}{dx} = y - x$ such that $y = 2$ when $x = 0$. 06 L_3 CO5

OR

b) Solve $\frac{dy}{dx} = x + 3y$ with $x_0 = 0$, $y_0 = 1$ by Euler's modified formula for $x = 0.1$. Correct to four decimal places by taking $h = 0.05$. 06 L_3 CO5

c) Find the value of $y(0.2)$ for the initial value problem $\frac{dy}{dx} = xy + y\sin x - 1$ with $y(0) = 1$, taking step size $h = 0.1$ using Adams –Bashforth-Moulton predictor- corrector method. 07 L_3 CO5

OR

c) Apply Runge-Kutta method of fourth order to find $\frac{dy}{dx} = x + y^2$ with initial conditions $y(0) = 1$ and find y at $x = 0.2$ in steps of $h = 0.1$. 07 L_3 CO5

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