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Sanjay Ghodawat University, Kolhapur

Established as State Private University under Govt. of Maharashtra. Act No XL, 2017

2018-19
EXM/P/09/00

Year and Program:
2018-19 M. Sc. I
Course Code – MTS 501

School of Science

Course Title – Linear Algebra

Department of Mathematics
Semester – I

Day and Date – Monday 26 Nov 18
PRN/ Exam Seat Number:

Student's Signature

Examination: (ESE)
End Semester Examination

Time: 10:00 to 1:00 pm
Max Marks: 100
Answer Booklet No.

Invigilator Signature

Instructions:

1) All questions are compulsory.
2) **Attempt Q.1 within first 30 minutes.**
3) Each MCQ type question has four choices out of which only one is the correct
4) Tick mark (√) the correct alternative which should be answered in question paper itself and submit to the Jr. Supervisor.
5) If you tick more than one option it will not be evaluated
6) Figures to the right indicate full marks
7) Use **Blue ball pen** only

Q.1	**Tick the correct answers.**	Marks	Bloom's Level	CO
i)	Which of the following vectors from C (R) lies in a linear span of $\{1+i\}$? A)1+4i B) 13+13i C) 9+11i D) all of above	02	L_2	CO1
ii)	Which of the following is not a subspace of C (R)? A) R(R) B) C (Q) C) {0}(R) D) C (R)	02	L_2	CO1
iii)	If V and U be a vector space of dimension 4 and 6 respectively then dim Hom (V,U) is_____ A) 24 B) 16 C) 36 D) 10	02	L_1	CO2
iv)	dim F(F)= _____ A)1 B)0 C)2 D) 11	02	L_1	CO2

v) If characteristic equation of matrix A is $c(x) = (x-11)^5$ and minimum polynomial of matrix is $m(x) = (x-11)^3$ then the Jordan canonical form of A is 02 L_3 CO3

A) $\begin{bmatrix} 11 & 1 & 0 & 0 & 0 \\ 0 & 11 & 1 & 0 & 0 \\ 0 & 0 & 11 & 0 & 0 \\ 0 & 0 & 0 & 11 & 0 \\ 0 & 0 & 0 & 0 & 11 \end{bmatrix}$ B) $\begin{bmatrix} 11 & 1 & 0 & 0 & 0 \\ 0 & 11 & 1 & 0 & 0 \\ 0 & 0 & 11 & 1 & 0 \\ 0 & 0 & 0 & 11 & 0 \\ 0 & 0 & 0 & 0 & 11 \end{bmatrix}$

C) $\begin{bmatrix} 11 & 1 & 1 & 0 & 0 \\ 0 & 11 & 1 & 0 & 0 \\ 0 & 0 & 11 & 0 & 0 \\ 0 & 0 & 0 & 11 & 0 \\ 0 & 0 & 0 & 0 & 11 \end{bmatrix}$ D) $\begin{bmatrix} 11 & 0 & 0 & 0 & 0 \\ 0 & 11 & 1 & 0 & 0 \\ 0 & 0 & 11 & 0 & 0 \\ 0 & 0 & 0 & 11 & 0 \\ 0 & 0 & 0 & 0 & 11 \end{bmatrix}$

vi) If $A = \begin{bmatrix} 1 & 0 & 7 \\ 2 & 5 & -1 \\ 3 & 4 & 5 \end{bmatrix}$ then Trace of 5A is _____ 02 L_2 CO4

A) 11 B) 55 C) 44 D) 5

vii) If $A = \begin{bmatrix} 1 & 225 & 20 \\ 0 & 25 & 5 \\ 0 & 0 & 4 \end{bmatrix}$ then det A^{-1} is _____ 02 L_2 CO4

A) 0.1 B) 0.01 C) 101 D)100

viii) Let V be a vector space of dimension 3 over a field F then dim B(V) is _____ (where B(V) denote the set of all bilinear form on V) 02 L_1 CO5

A) 9 B) 4 C) 6 D) 27

ix) Denote the matrix of the quadratic form $3x^2+4xy-y^2+8xz-6yz+z^2$ by A. then the trace of A is _____ 02 L_2 CO5

A) 5 B) 4 C) 3 D) 0

x) Which of the following matrix is not diagonalizable? 02 L_3 CO3

A) $\begin{bmatrix} -1 & 4 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 5 \end{bmatrix}$

B) $\begin{bmatrix} 2 & 1 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 3 \end{bmatrix}$

C) $\begin{bmatrix} 2 & 0 & 0 \\ 0 & 5 & 0 \\ 0 & 1 & 3 \end{bmatrix}$

D) $\begin{bmatrix} 1 & 4 & 5 \\ 0 & 2 & 6 \\ 0 & 0 & 3 \end{bmatrix}$

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Day and Date 26 NOV 18 | **End Semester Examination** | **Time:** 10:00 to 1:00pm | **Max Marks: 100**

Instructions:
1) All questions are compulsory.
2)Use of non-programmable calculator is allowed
3)Figures to the right indicate full marks.

		Marks	Bloom's Level	CO
Q.2	**Solve the following.**			
a)	If $S=\{v_1,v_2,\cdots,v_n\}$ is a basis of vector space $V(F)$ then show that every element of V can be expressed uniquely as a linear combination of $v_1,v_2,\cdots,v_n$.	06	L_3	CO1
	OR			
a)	Prove that V (F) be a vector space. A minimal generating set of V is a basis of V and conversely every basis of V is a minimal generating set of V.	06	L_3	CO1
b)	Let V is a FDVS and $\{v_1,v_2,\cdots,v_n\}$ is a LI subset of V then show that it can be extended to form a basis of V.	04	L_2	CO1
	OR			
b)	If S_1 and S_2 are subsets of a vector space $V(F)$ then prove that $S_1 \subseteq S_2 \Rightarrow L(S_1) \subseteq L(S_2)$ and $L(S_1 \cup S_2) = L(S_1) + L(S_2)$.	04	L_2	CO1
Q.3	**Solve the following.**			
a)	Let $V_1,V_2,...,V_m$ be vector spaces over a field F and $\dim(V_1 \oplus V_2 \oplus \cdots \oplus V_m) = \dim V_1 + \dim V_2 + \cdots + \dim V_m$ then prove that $V = V_1 \oplus V_2 \oplus \cdots \oplus V_m$ is finite dimensional iff each V_i is finite dimensional.	10	L_4	CO2

OR

a)	Let V be F.D.V.S. of dimension n over the field F. If $v \neq 0$ vector in V, then prove that $\exists$ a linear functional f on V such that $f(v) \neq 0$ Also, if V and V' are vector spaces over same field F, then show that $\dim V = \dim(\ker T) + \dim(\operatorname{Im} T)$	10	L_4	CO2

Q.4 **Solve the following.**

a)	If $T, S \in A(V)$ and S is regular then show that T and STS^{-1} have same minimal polynomial	06	L_3	CO3

OR

a)	If $\lambda \in F$ is a characteristic root of $T \in A(V)$ where V is FDVS, then show that λ is a root of minimal polynomial of T and T has only a finite number of characteristic root in F.	06	L_4	CO3
b)	Prove that the Relation on A(V) defined by similarity is an equivalence relation	04	L_2	CO3

OR

b)	Prove that Let V be a FDVS over F and let T be a linear operator on V Then a scalar λ in F is an eigenvalues of T iff $\det(T-\lambda I)=0$	04	L_2	CO3

Q.5

A **Attempt Any One**

a)	Define self adjoint or Hermitian and show that the Hermitian linear tranformation T is non negative iff all of it's characteristic roots are non negative	09	L_4	CO4
b)	If $T \in A(V)$ then show that $T^* \in A(V)$, also show that *i)* $(T^*)^* = T$ ii) $(S+T)^* = S^* + T^*$ iii) $(\lambda S)^* = \bar{\lambda} S^*$	09	L_3	CO4

B	**Attempt Any two**			
a)	Prove the linear transformation T on V is unitary iff it takes an orthonormal basis of V into an orthonormal basis of V	08	L_4	CO4
b)	Prove if $T \in A(V)$ such that $(T(v), v) = 0$ for all $v \in V$ then T=0 Also if (T(v), T(v))=(v,v) for all $v \in V$ then prove that T is unitary	08	L_3	CO4
c)	If N is a normal linear trasformation and for $\lambda \in F$ $(N-\lambda I)^k(v)=0$ then prove that $N(v)=\lambda v$ Also prove that if N is a normal AN=NA, then $AN^*=N^*A$	08	L_3	CO4
Q.6				
A	Define matrix of f in bilinear form. Prove that any n dimensional vector space V over F and any ordered basis β for V, $\psi_\beta : B(V) \to M_{n\times n}(F)$ is an isomorphim.	10	L_4	CO5
B	Let $u = (x_1, x_2, x_3)$ and $v = (y_1, y_2, y_3)$ and let $f(u,v) = 3x_1y_1 - 2x_1y_2 + 5x_2y_1 + 7x_2y_2 - 8x_2y_3 + 4x_3y_2 - x_3y_3$ Express f in matrix notation	03	L_2	CO5
C	**Attempt Any Two**			
a)	If V is a vector space over F and $T : V \to V$ is a linear transformation on V. Suppose f is a bilinear form on V then show that g is a bilinear form from $V \times V$ into F defined as $g(\alpha, \beta) = f(T(\alpha), T(\beta))$	06	L_3	CO5
b)	Define symmetric and Skew- symmetric bilinear form and prove that for any n-dimensional vector space V over F, B(V) has dimension n^2	06	L_2	CO5
c)	Let $T_1 : R^2(R) \to R$ be linear functional given by $T_1(x, y) = 2x + 3y$ and $T_2 : R^3(R) \to R$ be linear functional given by $T_2(x, y, z) = 2x - y + 3z$ and $f : R^2 \times R^3 \to R$ is bilinear form given by $f(\alpha, \beta) = T_1(\alpha)T_2(\beta)$ then find i) Rank of f ii) f((1,2)(3,4,5))	06	L_5	CO5

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